Rovnice a nerovnice s kombinačními čísly

① Řešte v $\mathcal{N}$

$\binom{n}{k}$ defin. pro $n \geq k,\ n \in \mathcal{N}_0, k \in \mathcal{N}_0$

a) $\binom{n}{2} + \binom{n-1}{2} = 4$

$$\left[\begin{array}{c} \frac{n!}{(n-2)!\cdot 2!} + \frac{(n-1)!}{[(n-1)-2)]!\cdot 2!} = 4 \\ \frac{n\cdot(n-1)\cdot(n-2)!}{(n-2)!\cdot 2!} + \frac{(n-1)\cdot(n-2)\cdot(n-3)!}{(n-3)!\cdot 2!} = 4 \end{array}\right]$$

$$\frac{n(n-1)}{2\cdot 1} + \frac{(n-1)(n-2)}{2\cdot 1} = 4$$

$$n^2 - n + n^2 - 2n - n + 2 = 8$$

$$2n^2 - 4n - 6 = 0$$

$$\left[\begin{array}{cc} n \geq 2 & n-1 \geq 2 \\ & n \geq 3 \end{array}\right]$$

$D = \mathcal{N} - \{1, 2\}$

$$n^2 - 2n - 3 = 0$$

$$(n-3)(n+1) = 0$$

$n_1 = 3_{\in D}$ $\quad n_2 = -1_{\notin D}$ nevyh.

$K = \{3\}$

b) $\binom{n}{3} + \binom{n+2}{3} + \binom{n+4}{3} = \frac{n^3}{2} + 88$

$$\left[\begin{array}{ccc} \binom{n}{k} & n \geq k & \\ n \geq 3 & n+2 \geq 3 & n+4 \geq 3 \\ & n \geq 1 & n \geq -1 \end{array}\right]$$

$D = \mathcal{N} - \{1, 2\}$

$$\frac{n(n-1)(n-2)}{3\cdot 2\cdot 1} + \frac{(n+2)(n+1)n}{3\cdot 2\cdot 1} + \frac{(n+4)(n+3)(n+2)}{3\cdot 2\cdot 1} = \frac{3n^3 + 6\cdot 88}{6}$$

$$n(n^2 - 3n + 2) + (n^2 + 3n + 2)n + (n^2 + 7n + 12)(n+2) = 3n^3 + 528$$

$$n^3 - 3n^2 + 2n + n^3 + 3n^2 + 2n + n^3 + 7n^2 + 12n + 2n^2 + 14n + 24 = 3n^3 + 528$$

$$9n^2 + 30n - 504 = 0$$

$$3n^2 + 10n - 168 = 0$$

$$n_{1,2} = \frac{-10 \pm \sqrt{100 + 4\cdot 3\cdot 168}}{6} = \frac{-10 \pm \sqrt{2\,116}}{6} = \frac{-10 \pm 46}{6}$$

$n_1 = 6_{\in D}$ $\quad n_2 = -\frac{56}{6}{}_{\notin D}$ nevyh. $\quad K = \{6\}$

② Řešte v $\mathcal{N}_0$

$$\binom{n}{n-k} = \binom{n}{k}$$

a) $\binom{x}{x-2} - \binom{x+1}{x} = 4$

$$\left[\binom{x}{x-(x-2)} - \binom{x+1}{x+1-x} = 4\right]$$

$$\binom{x}{2} - \binom{x+1}{1} = 4$$

$$\frac{x(x-1)}{2\cdot 1} - \frac{x+1}{1} = 4 \quad |\cdot 2$$

$$x^2 - x - 2(x+1) = 8$$

$$x^2 - 3x - 10 = 0$$

$$\left[\begin{array}{cccc} x \in \mathcal{N}_0 & x \geq x-2 & x-2 \geq 0 & x+1 \geq x \\ & 0x \geq -2\ pl. & x \geq 2 & 0x \geq -1\ pl. \end{array}\right]$$

$D = \mathcal{N} - \{1\}$

$$(x-5)(x+2) = 0$$

$x_1 = 5_{\in D}$ $\quad x_2 = -2_{\notin D}$ nevyh. $\quad K = \{5\}$

b) $\binom{x}{x-2} + \binom{x}{x-1} = \binom{x+1}{2}$

$$\binom{x}{2} + \binom{x}{1} = \binom{x+1}{2}$$

$$\frac{x\cdot(x-1)}{2\cdot 1} + \frac{x}{1} = \frac{(x+1)\cdot x}{2\cdot 1} \quad |\cdot 2$$

$$x^2 - x + 2x = x^2 + x$$

$$0x = 0 \quad \text{pl.}$$

$$\left[\begin{array}{ccc} x \in \mathcal{N}_0 & x-2 \geq 0 & x-1 \geq 0 \\ & x \geq 2 & x \geq 1 \end{array}\right]$$

$D = \mathcal{N} - \{1\}$

$K = D = \mathcal{N} - \{1\}$

c) $2\cdot\binom{x+6}{x+4}-\binom{x+4}{x+2}=4!+\binom{5}{2}\cdot x$

$$\begin{bmatrix} x\in\mathcal{N}_0 & x+6\geq 0 & x+4\geq 0 \\ & x\geq -6 & x\geq -4 \end{bmatrix}$$

$$D=\mathcal{N}_0$$

$$2\cdot\binom{x+6}{2}-\binom{x+4}{2}=4\cdot 3\cdot 2\cdot 1+\frac{5\cdot 4}{2\cdot 1}\cdot x$$

$$\frac{2(x+6)(x+5)}{2\cdot 1}-\frac{(x+4)(x+3)}{2\cdot 1}=24+10x \qquad |\cdot 2$$

$$2(x^2+11x+30)-(x^2+7x+12)=48+20x$$

$$2x^2+22x+60-x^2-7x-12=48+20x$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x_1=0_{\in D}\quad x_2=5_{\in D} \qquad\qquad K=\{0,5\}$$

d) $K(n-3,n)=20$

$[n\geq 3 \quad n\in\mathcal{N}_0]$ $\qquad D=\mathcal{N}-\{1,2\}$

$$\binom{n}{n-3}=20$$

$$\binom{n}{3}=20$$

$$\frac{n(n-1)(n-2)}{3\cdot 2\cdot 1}=20$$

[**2. zp.** $n(n^2-3n+2)=120$ rce (∗)
$n^3-3n^2+2n-120=0$ rovnice 3. stupně
⇒ zkusmo 1. kořen n_1 ⇒ rovnice 2. stupně
$(n-n_1)\cdot\Big(\underbrace{an^2+bn+c}_{\text{kv. rov.}}\Big)=0$
⇒ lépe řešit rci (∗) ZKUSMO]

(∗) $n(n-1)(n-2)=120$

⇒ ZKUSMO $120=6\cdot 20=6\cdot 5\cdot 4=6\cdot(6-1)\cdot(6-2)$ ⇒ $n=6$ $\qquad K=\{6\}$

③ Řešte v $\mathcal{N}$

a) $K(x-2,x)<45$

$[x\geq 2 \quad x\in\mathcal{N}]$
$D=\mathcal{N}-\{1\}$

$$\binom{x}{x-2}<45$$

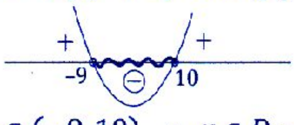
$$\binom{x}{2}<45$$

$$\frac{x(x-1)}{2\cdot 1}<45$$

$$x^2-x<90$$

$$x^2-x-90<0$$

$(x-10)(x+9)<0$ $\quad [x_0=10, x_0=-9]$



$x\in(-9,10) \ \wedge\ x\in D=\mathcal{N}-\{1\}$
$K=\{2,3,4,5,6,7,8,9\}$

b) $\binom{n+1}{2}+\binom{n+4}{2}+\binom{n+7}{2}<90$

$$\begin{bmatrix} n+1\geq 2 & n+4\geq 2 & n+7\geq 2 & n\in\mathcal{N} \\ n\geq 1 & n\geq -2 & n\geq -5 & \end{bmatrix}$$

$$D=\mathcal{N}$$

$$\frac{(n+1)n}{2\cdot 1}+\frac{(n+4)(n+3)}{2\cdot 1}+\frac{(n+7)(n+6)}{2\cdot 1}<90$$

$$n^2+n+n^2+7n+12+n^2+13n+42-180<0$$

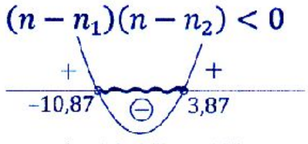
$$3n^2+21n-126<0$$

$$n^2+7n-42<0$$

$$n_{1,2}=\frac{-7\pm\sqrt{49+4\cdot 42}}{2}=\frac{-7\pm\sqrt{217}}{2}\doteq\frac{-7\pm 14{,}73}{2}$$

$$n_1\doteq\frac{7{,}73}{2}\doteq 3{,}87 \qquad n_2\doteq-\frac{21{,}73}{2}\doteq -10{,}87$$

$(n-n_1)(n-n_2)<0$



$n\in(-10{,}87; 3{,}87)\wedge\ n\in\mathcal{N}$

$K=\{1,2,3\}$